Hong Kong Mathematics Olympiad (2003 – 2004)
Heat Event (Group)
香港数学竞赛 (2003 – 2004)
初赛项目(团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 若 $x=\frac{1}{2}+\left(\frac{1}{3}+\frac{2}{3}\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right)+\left(\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}\right)+\cdots+\left(\frac{1}{100}+\frac{2}{100}+\cdots+\frac{99}{100}\right)$，求 x 的值。

   If $x=\frac{1}{2}+\left(\frac{1}{3}+\frac{2}{3}\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right)+\left(\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}\right)+\cdots+\left(\frac{1}{100}+\frac{2}{100}+\cdots+\frac{99}{100}\right)$, find the value of x.

2. 若 z 是方程 $6\times4^x-13\times6^x+6\times9^x=0$ 的正数根，求 z 的值。

   If z is the positive root of the equation $6\times4^x-13\times6^x+6\times9^x=0$, find the value of z.

3. 若最多有 k 个互不全等的登腰三角形，其周界为 25 cm 及其三边的长度以 cm 表示时均为正整数，求 k 的值。

   If there are at most k mutually non-congruent isosceles triangles whose perimeter is 25 cm and the lengths of the three sides are positive integers when expressed in cm, find the value of k.

4. 已知 a、b 为实数并且满足 $a^3=2004$ 及 $b^2=2004$。若满足不等式 $a<x<b$ 的整数 x 有 h 个，求 h 的值。

   Given that a, b are positive real numbers satisfying $a^3=2004$ and $b^2=2004$. If the number of integers x that satisfy the inequality $a<x<b$ is h, find the value of h.

5. 若 R 个连续正整数之和是 1000 (其中 $R>1$)，求 R 的最小值。

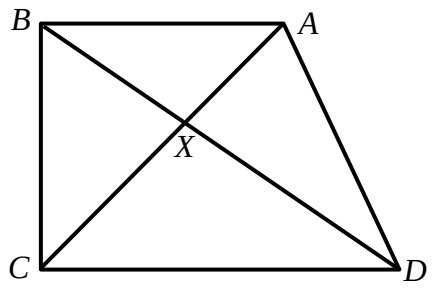
   If the sum of R consecutive integers is 1000 (where $R>1$), find the least value of R.

6. 若 a 、 b 及 c 是正整数且 $abc+ab+bc+ca+a+b+c=2003$，求 abc 的最小值。

If a, b and c are positive integers such that $abc+ab+bc+ca+a+b+c=2003$, find the least value of abc.

7. 在图中， $ABCD$ 是梯形，AB、 CD 垂直于 BC，对角线 AC 和 BD 相交于 X。若 $AB=9$ cm，$BC=12$ cm，$CD=16$ cm，ΔBXC 的面积为 W cm^2，求 W 的值。

In the figure, $ABCD$ is a trapezium, the segments AB and CD are both perpendicular to BC and the diagonals AC and BD intersect at X. If $AB=9$ cm, $BC=12$ cm and $CD=16$ cm, and the area of ΔBXC is W cm^2, find the value of W.

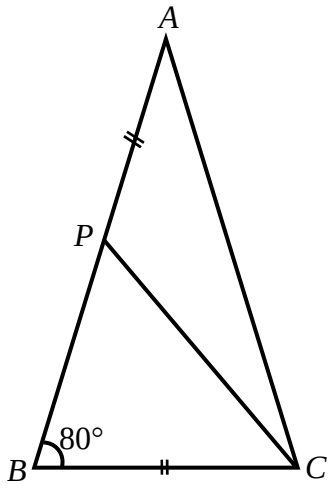


8. 设 $y=\log_{1400}\sqrt{2}+\log_{1400}\sqrt[3]{5}+\log_{1400}\sqrt[6]{7}$，求 y 的值。

Let $y=\log_{1400}\sqrt{2}+\log_{1400}\sqrt[3]{5}+\log_{1400}\sqrt[6]{7}$, find the value of y.

9. 在图中，ΔABC 是等腰三角形，$AB = AC$ 及 $\angle ABC = 80°$。若 P 是 AB 上一点使得 $AP = BC$，$\angle ACP = k°$，求 k 的值。

In the figure , ΔABC is an isosceles triangle with $AB = AC$ and $\angle ABC = 80°$. If P is a point on AB such that $AP = BC$, $\angle ACP = k°$, find the value of k .



10. 若点 $P(a, b)$ 在直线 $x - y + 1 = 0$ 上使得点 P 与点 $A(1, 0)$ 之间的距离和点 P 与点 $B(3, 0)$ 之间的距离之和为最小，求 $a + b$ 的值。

Suppose $P(a, b)$ is a point on the straight line $x - y + 1 = 0$ such that the sum of the distance between P and the point $A(1, 0)$ and distance between P and the point $B(3, 0)$ is the least , find the value of $a + b$.